Maths

# NORTH SYDNEY BOYS HIGH SCHOOL

## 2017 HSC ASSESSMENT TASK 3 (TRIAL HSC)

# Mathematics

### General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- For Section I, shade the correct box on the sheet provided
- For Section II, write in the booklet provided
- Each new question is to be started on a **new page.**
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question

- Attempt all questions

**Class Teacher:**
(Please tick or highlight)

- O Mr Zuber
- O Mr Ireland
- O Mr Berry
- O Ms Ziaziaris
- O Mr Hwang

Student Number

(To be used by the exam markers only.)

| Question No | 1-10 | 11 | 12 | 13 | 14 | 15 | 16 | Total | Total |
|---|---|---|---|---|---|---|---|---|---|
| Mark | /10 | /15 | /15 | /15 | /15 | /15 | /15 | /100 | /100 |

## Section I

**10 marks**
**Attempt Questions 1-10**
**Allow about 15 minutes for this section.**

Use the multiple-choice answer sheet for Questions 1-10

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**1** $x + 2y - (x - 2z) - (2y + x - (2z - x))$ simplifies to

(A) $4z + 2x$

(B) $4z - 2x$

(C) $4y + 4z - 2x$

(D) $4y - 4z - 2x$

**2** Which of the following **best** describe the locus of a point which moves so as to always be equidistant from two fixed points?

(A) a circle

(B) a parabola

(C) a straight line

(D) an exponential

**3** $\log_3 15 + \log_3 18 - \log_3 10$ evaluates to:

(A) 1

(B) 2

(C) 3

(D) 0

**4** Find the limiting sum of the following geometric series:

$$-1+\frac{2}{3}-\frac{4}{9}+\frac{8}{27}+\cdots$$

(A) $-3$

(B) $3$

(C) $\frac{3}{5}$

(D) $-\frac{3}{5}$

**5** Which of the following most accurately describe the parabola $y=-2x^2+9x-11$?

(A) Positive definite

(B) Negative Definite

(C) Indefinite

(D) None of the above

**6** The centre of the circle whose equation is $x^2-6x+y^2+4y-3=0$ is

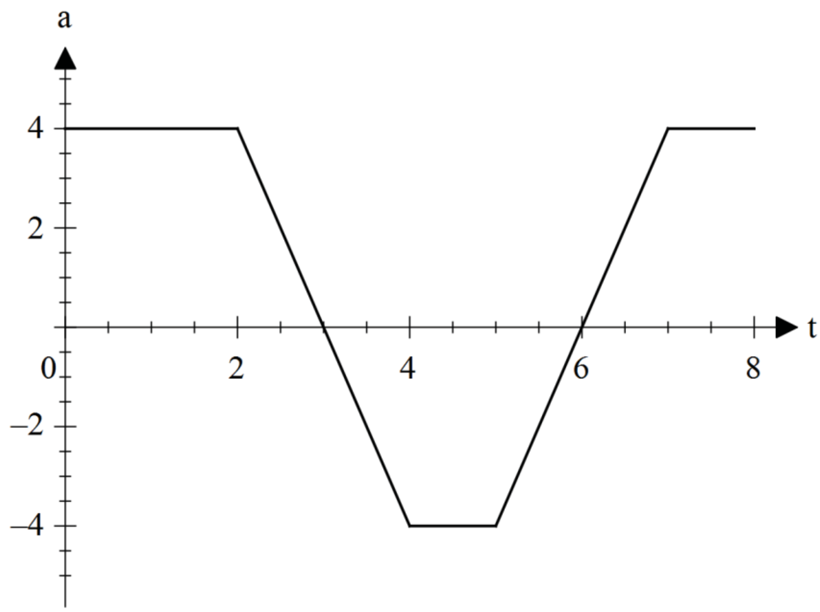
(A) (6, -4)

(B) (-3, 2)

(C) (0, 0)

(D) (3, -2)

**7** The acceleration – time graph of a particle is shown below:




The time(s) when the particle has an absolute minimum velocity is

(A) $4 < t < 5$

(B) $t = 0$

(C) $t = 3$

(D) $t = 6$

**8** What is the value of $\int_{-2}^{3} |x - 1| dx$ ?

(A) $\frac{11}{2}$

(B) $\frac{5}{2}$

(C) $\frac{13}{2}$

(D) $\frac{17}{2}$

**9** Which of the following is **not** equivalent to $\int_{-a}^{a} f(x)dx$ when $f(x)$ is **any** odd function?

(A) 0

(B) $3\int_{-a}^{a} f(x)dx$

(C) $\int_{-a}^{0} f(x)dx + \left|\int_{0}^{a} f(x)dx\right|$

(D) $\int_{-a}^{0} f(x)dx - \int_{a}^{0} f(x)dx$

**10** How many non-similar triangles have angles whose degree measures are distinct positive integers in arithmetic progression?

(A) 59

(B) 60

(C) 89

(D) 178

## Section II

**90 marks**
**Attempt Questions 11-16**
**Allow about 2 hours and 45 minutes for this section.**

Answer each question on a NEW page. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Start a NEW page in your writing booklet.

(a) Express $\dfrac{1}{\sqrt{5}-\sqrt{2}}$ with a rational denominator **2**

(b) Solve $|x-1| < 3$ and graph the solution on a number line. **2**

(c) The arc of a circle subtends an angle of $100°$ at the centre. If the radius of the circle is 12cm, calculate the exact length of the arc. **2**

(d) Differentiate the following:

(i) $\tan 2x$ **1**

(ii) $x^2 \log_e x$ **2**

(e) Find the following integral:

$$\int x^3 + \frac{1}{2x}\, dx$$ **2**

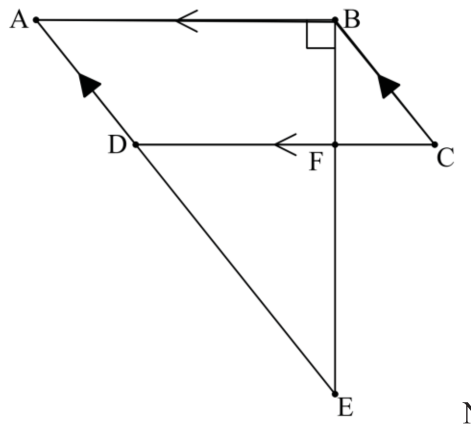
(f) Evaluate the following integral

(i) $\displaystyle\int_0^{\frac{2\pi}{3}} \sin\left(\frac{x}{2}\right) dx$ **2**

(ii) $\displaystyle\int_1^2 e^{4x} + e^{-x}\, dx$ **2**

**Question 12** (15 marks) Start a NEW page in your writing booklet.

(a)



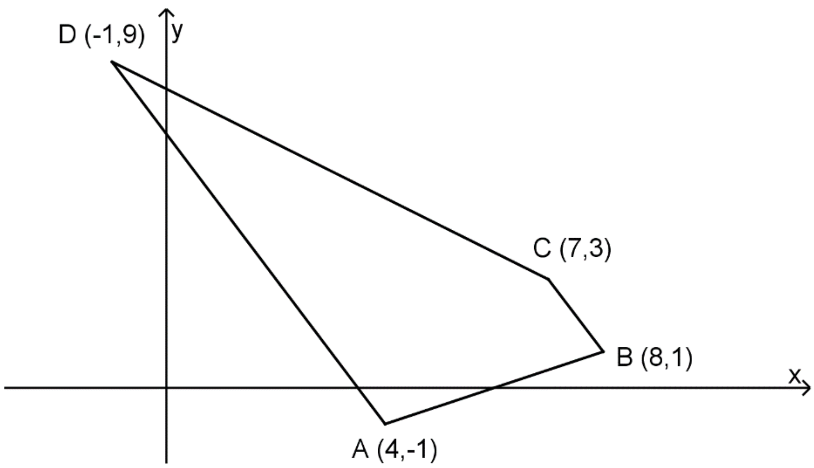

NOT TO SCALE

$ABCD$ is a parallelogram where $FB \perp AB$.

(i) Prove that $\Delta CBF \mathbin{|||} \Delta AEB$ **3**

(ii) If $CF = 3$cm and $BC = 7$cm and $AE = 15$ cm, find the length of $AB$ **2**

(b)




NOT TO SCALE

$A, B, C$ and $D$ are the points $(4, -1)$ , $(8, 1)$, $(7, 3)$ and $(-1, 9)$ respectively.

(i) Show the equation of $AC$ is $4x - 3y - 19 = 0$ **2**

(ii) Show $BC \parallel AD$ **1**

(iii) Show $\angle ACD = 90°$ **1**

(iv) Show the length of $AC$ is 5 units. **1**

(v) Find the perpendicular distance of $B$ from $AC$ **2**

(vi) Find the area of the trapezium $ABCD$ **3**

**Question 13** (15 marks) Start a NEW page in your writing booklet.

(a) The equation of a parabola is $x^2 = 12y + 6x + 15$. Find

(i) the coordinates of the vertex **2**

(ii) the coordinates of the focus **1**

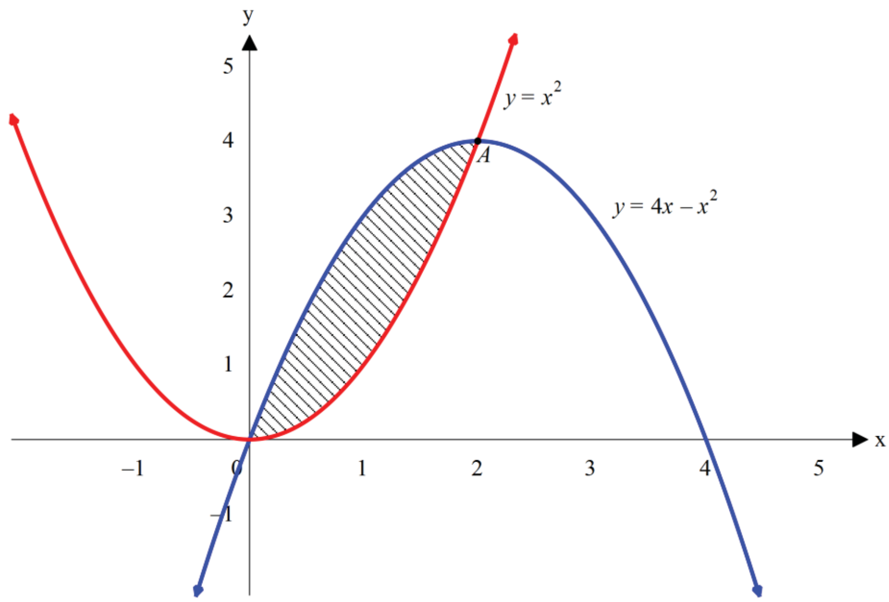
(iii) the equation of the directrix **1**

(b) For what values of $p$ does the equation $x^2 - px + p - 1 = 0$ have

(i) equal roots. **2**

(ii) one of the roots equal to 3 **2**

(c) The diagram shows the curves $y = x^2$ and $y = 4x - x^2$, which intersect at the origin and at the point $A$.




(i) Show that the coordinates of $A$ are (2, 4) **2**

(ii) Hence find the area enclosed between the curves. **3**

(d) Using the trapezoidal rule with 4 subintervals, evaluate the area under the curve $y = x^x$ between $x = 1$ and $x = 3$, correct to 2 decimal places. **2**

**Question 14** (15 marks) Start a NEW page in your writing booklet.

(a) A function is defined by $f(x) = \dfrac{x^3}{4}(x-8)$

(i) Find the coordinates of the stationary point(s) of the graph of $y = f(x)$ and determine their nature. **4**

(ii) Sketch the graph of $y = f(x)$ showing all its essential features including stationary points and intercepts. **2**

(iii) For what values of $x$ is the curve increasing? **1**

(b) A rainwater tank with a volume of $9\text{m}^3$ is installed in a new house. At 8am rain begins to fall and flows into the empty tank at the rate given by

$$\frac{dV}{dt} = \frac{36t}{t^2+20}$$

where $t$ is the time in hours and $V$ is the volume measured in cubic metres. ($t = 0$ is represented by 8am.)

(i) Show by integration or otherwise that the volume of water in the tank at time, $t$ is given by **2**

$$V = 18\log_e\left(\frac{t^2+20}{20}\right), t > 0$$

(ii) Find the time when the tank will be completely filled with water (to the nearest minute). **3**

(iii) Later, when the tank is full and the rain has stopped, Louise turns on the pump which pumps the water out at the rate given by **3**

$$\frac{dV}{dT} = \frac{T^2}{k}$$

where $T$ is the time from when Louise turns on the pump and $k$ is a constant. The pump continues for 5 hours until the tank is empty.

Find the value of $k$.

**Question 15** (15 marks) Start a NEW page in your writing booklet.

(a) A particle is moving along a straight line. Its displacement from a fixed point on the line at time $t$ seconds is given by $x = 4t^3 - 3t^2 - 18t + 1$, where $t \geq 0$ and $x$ is in metres.

(i) Find the velocity $v$ in terms of $t$. **1**

(ii) Find the acceleration, $a$, in terms of $t$ **1**

(iii) At what time(s) does the particle come to rest? **1**

(iv) Where does the particle come to rest? **1**

(v) How far does the particle travel in the first 2 seconds? **2**

(b) Two cultures of bacteria are prepared in a laboratory. They are to be used to test the effectiveness of two drugs. One culture has 1000 bacteria and Drug A reduces this number to 250 in 5 minutes. The other culture has 1250 bacteria and Drug B reduces this number to 500 in 3 minutes. Both cultures are reduced according to the model $N = N_0 e^{-kt}$ where $N$ is the number of bacteria and $t$ is the time since the drug was administered in minutes.

(i) Which drug is more effective in reducing the number of bacteria? Support your answer with calculations. **3**

(ii) How long will it take for Drug B to reduce the second culture to 10% of its original number of bacteria? **2**

(c) (i) Show that $\dfrac{d}{dx}\left(xe^{\frac{x}{2}}\right) = e^{\frac{x}{2}} + \dfrac{1}{2}xe^{\frac{x}{2}}$ **2**

(ii) Hence find $\displaystyle\int xe^{\frac{x}{2}}dx$ **2**

**Question 16** (15 marks) Start a NEW page in your writing booklet.

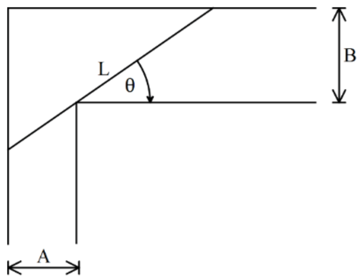
(a) Given $\tan A = \sqrt[3]{\frac{x}{y}}$ and $0 < A < \frac{\pi}{2}$

(i) Show that $\cos A = \frac{y^{\frac{1}{3}}}{\sqrt{x^{\frac{2}{3}} + y^{\frac{2}{3}}}}$ **2**

(ii) Hence write down the value of $\sin A$ **1**

(b) Two corridors meet at right angles. One has a width of $A$ metres, and the other has a width of $B$ metres.

Mario the plumber wants to find the length of longest pipe, $L$ that he can carry horizontally around the corner as seen in the diagram below. Assume that the pipe has negligible diameter.




(i) Show that $L = A \sec\theta + B \operatorname{cosec}\theta$ **1**

(ii) Explain why the length of the pipe, $L$, needs to be minimised in order to obtain the length of longest pipe that can be carried around the corner. **1**

(iii) Hence show that when $\tan\theta = \sqrt[3]{\frac{B}{A}}$ the solution will be minimised. [You do **NOT** need to test to show that the solution will give a minimum length.] **4**

(iv) Hence using the result in part (a), show that the length of the largest pipe that can be carried around the corner is $L = \left(A^{\frac{2}{3}} + B^{\frac{2}{3}}\right)^{\frac{3}{2}}$ **3**

(c) The sequence **3**

$$1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, ...$$

consists of 1's separated by blocks of 2's with $n$ $2's$ in the $n$th block.

Find the sum of the first 1234 terms in the sequence.

**End of Examination**

Multiple Choice

1. B  2. C  3. C  4. D  5. B  6. D  7. B  8. C  9. C  10. A

Question 11

a) $\frac{1}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

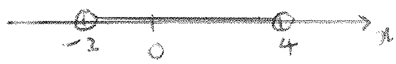
$= \frac{\sqrt{5}+2}{5-2}$

$= \frac{\sqrt{5}+\sqrt{2}}{3}$

b) $|x-1|<3$

$-3<x-1<3$

$-2<x<4$




c) $100^\circ = \frac{5\pi}{9}$ radians

$l = r\theta$

$= 12 \times \frac{5\pi}{9}$

$= \frac{20\pi}{3}$ cm

d) (i) $2\sec^2 2x$

(ii) $\frac{d}{dx}(x^2 \log_e x)$

$= x^2 \cdot \frac{1}{x} + \log_e x \cdot 2x$

$= x + 2x\log_e x$

$= x(1+2\log_e x)$

e) $\int x^3 + \frac{1}{2x}\, dx$

$= \frac{x^4}{4} + \frac{1}{2}\log_e|x| + c$

f) i) $\int_0^{\frac{2\pi}{3}} \sin\left(\frac{x}{2}\right) dx$

$= \left[-2\cos\frac{x}{2}\right]_0^{\frac{2\pi}{3}}$

$= -2\left[\cos\frac{\pi}{3} - \cos 0\right]$

$= -2\left[\frac{1}{2} - 1\right]$

$= 1$

ii) $\int_1^2 e^{4x} + e^{-x}\, dx$

$= \left[\frac{1}{4}e^{4x} - e^{-x}\right]_1^2$

$= \frac{1}{4}e^8 - \frac{1}{e^2} - \frac{1}{4}e^4 + \frac{1}{e}$

Question 12.

a) i) In $\triangle CBF$ and $\triangle AEB$

$\angle CBF = \angle BEA$ (alternate angles on parallel lines are equal, $AE \parallel BC$)

$\angle BCF = \angle EAB$ (opposite angles of parallelogram are equal)

$\angle ABF = \angle BFC$ (alternate angles on parallel lines are equal, $AB \parallel DC$)

use two

$\therefore \triangle CBF \parallel\!| \triangle AEB$ (equiangular)

ii) $\frac{AB}{AE} = \frac{CF}{CB}$ (corresponding sides of similar triangles are in proportion)

$\frac{AB}{15} = \frac{3}{7}$

$AB = 6\frac{3}{7}$ cm

b) i) $m_{AC} = \frac{4}{3}$

$\therefore$ equation of AC: $y - 3 = \frac{4}{3}(x - 7)$

$3y - 9 = 4x - 28$

$4x - 3y - 19 = 0$

ii) $m_{BC} = \frac{2}{-1} = -2$

$m_{AD} = \frac{9-(-1)}{-1-4}$

$= \frac{10}{-5}$

$= -2$

Since $m_{BC} = m_{AD}$ then $BC \parallel AD$

iii) $m_{DC} = \dfrac{9-3}{-1-7}$

$= -\dfrac{3}{4}$

Since $m_{AC} \times m_{DC} = \dfrac{4}{3} \times -\dfrac{3}{4}$

$= -1$

$AC \perp DC$

iv) $AC = \sqrt{(7-4)^2 + (3-(-1))^2}$

$= \sqrt{9+16}$

$= 5$

v) $d = \left| \dfrac{4\times 8 - 3\times 1 - 19}{\sqrt{4^2 - (-3)^2}} \right|$

$= 2$

vi) $DC = \sqrt{8^2 + 6^2}$

$= 10$

Area of trapezium = Area of $\triangle ACD$ + Area of $\triangle ACB$

$= \frac{1}{2} AC \times DC + \frac{1}{2} AC \times d$

$= \frac{5}{2}(10+2)$

$= 30 \ u^2$

Question 13

a) (i) $x^2 = 12y + 6x + 15$

$x^2 - 6x = 12y + 15$

$(x-3)^2 = 12y + 24$

$= 12(y+2)$

$\therefore$ vertex $(3, -2)$

(ii) Focal length: $a = 3$

$\therefore$ Focus: $(3, 1)$

(iii) $y = -5$

b) (i) Equal roots when $\Delta = 0$

$\Delta = (-p)^2 - 4(p-1)$

$= p^2 - 4p + 4$

$= (p-2)^2$

$\Delta = 0 \Rightarrow p = 2$

(ii) when $x = 3$ is a root

then $9 - 3p + p - 1 = 0$

$8 - 2p = 0$

$2p = 8$

$p = 4$.

c) i) Solving the equations simultaneously

$x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$\therefore x = 0$ or $x = 2$

$\therefore$ the $x$ coordinate of A is 2.

when $x = 2$ $y = 2^2 = 4$

$\therefore$ A (2, 4)

(ii) $A = \int_0^2 (4x - x^2) - x^2 \, dx$

$= \int_0^2 4x - 2x^2 \, dx$

$= \left[ 2x^2 - \frac{2x^3}{3} \right]_0^2$

$= \left( 2 \times 4 - \frac{2 \times 8}{3} \right)$

$= \frac{8}{3} \, u^2$

d)

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| $y$ | 1 | 1.84 | 4 | 9.88 | 27 |

$A \doteqdot \frac{0.5}{2} \left[ 1 + 3^3 + 2\left(1.5^{1.5} + 2^2 + 2.5^{2.5}\right) \right]$

$\doteqdot 14.86$ (to 2 decimal place)

Question 14

a) i) $f(x) = \frac{x^3}{4}(x-8)$

$= \frac{x^4}{4} - 2x^3$

$f'(x) = x^3 - 6x^2$

$= x^2(x-6)$

Stationary points occur when $f'(x) = 0$

$\therefore x^2(x-6) = 0$

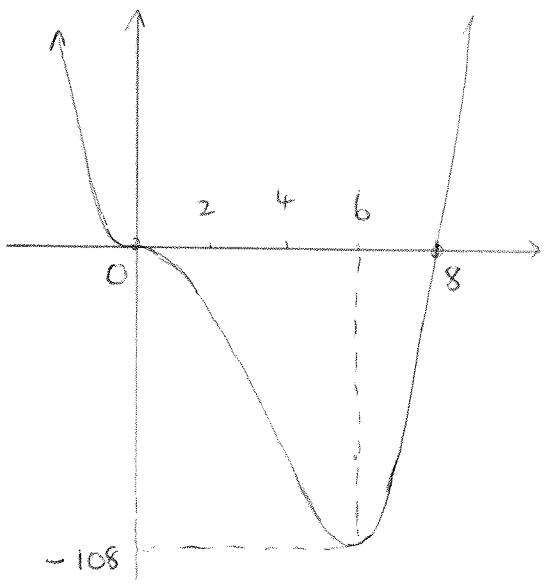
$\therefore x = 0$ OR $x = 6$.

| $x$ | $-1$ | $0$ | $5$ | $6$ | $7$ |
|---|---|---|---|---|---|
| $f'(x)$ | $-7$ | $0$ | $-25$ | $0$ | $49$ |
| | \ | — | \ | — | / |

$\therefore$ horizontal point of inflexion at $(0,0)$

and a absolute minimum turning point at $(6, -108)$

ii)




iii) Increasing when $x > 6$.

b) i) $\frac{dV}{dt} = \frac{36t}{t^2+20}$

if students differentiate $V$ to get $\frac{dV}{dt}$ they must test initial condition satisfies.

$V = \int \frac{36t}{t^2+20}\,dt$

$= 18\int \frac{2t}{t^2+20}\,dt$

$= 18\log_e(t^2+20) + C$

when $t=0$ $V=0$

$\therefore C = -18\log_e 20$

$\therefore V = 18\log_e(t^2+20) - 18\log_e 20$

$= 18\log_e\left(\frac{t^2+20}{20}\right)$

ii) Tank will be filled when $V=9$

$9 = 18\log_e\left(\frac{t^2+20}{20}\right)$

$\frac{1}{2} = \log_e\left(\frac{t^2+20}{20}\right)$

$\frac{t^2+20}{20} = e^{\frac{1}{2}}$

$t^2 = 20e^{\frac{1}{2}} - 20$

$t \doteqdot \sqrt{12.9744}$ as $t>0$

$\doteqdot 3.6$ hours or 3 hrs and 37 min.

iii) $V = \int \frac{T^2}{k}\,dT$

$V = \frac{T^3}{3k} + D$

when $T=0$ $V=9$ $\therefore D=9$

$V = \frac{T^3}{3k} + 9$

when $T=5$ $V=0$

$\therefore 0 = \frac{5^3}{3k} + 9$

$\frac{125}{3k} = -9$

$k = \frac{-125}{27}$

Question 15

a) i) $v = 12t^2 - 6t - 18$

ii) $a = 24t - 6$

iii) $12t^2 - 6t - 18 = 0$

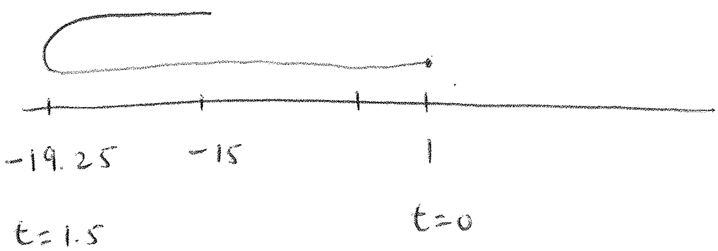
$6(2t^2 - t - 3) = 0$

$6(2t-3)(t+1) = 0$

$\therefore t = 1.5$ as $t \geq 0$

iv) $-19.25$ ( 19.5m left of 0 )

v)




when $t = 0$ $x = 1$

when $t = 1.5$ $x = -19.25$

when $t = 2$ $x = -15$

$\therefore$ total distance travelled :

$20.25 + 4.22$

$= 24.5$m

b) i) Drug A

$250 = 1000 e^{-5k}$

$0.25 = e^{-5k}$

$-5k = \ln(0.25)$

$k = \dfrac{\ln(0.25)}{-5}$

$\doteqdot 0.277\ldots$

Drug B

$500 = 1250 e^{-3k}$

$0.4 = e^{-3k}$

$-3k = \ln(0.4)$

$k = \dfrac{\ln(0.4)}{-3}$

$\doteqdot 0.305$

The decay constant for drug B is larger than the one for drug A $\therefore$ drug B is more effective.

ii) $0.1 = e^{-0.305t}$

$\ln(0.1) = -0.305t$

$t = \dfrac{\ln(0.1)}{-0.305}$

$\doteqdot 7.54$ minutes.

c) i) $\dfrac{d}{dx}\left(xe^{\frac{x}{2}}\right) = \overbrace{e^{\frac{x}{2}} \cdot 1}^{vu'} + \overbrace{x \cdot \frac{1}{2} \cdot e^{\frac{x}{2}}}^{uv'}$

$= e^{\frac{x}{2}} + \frac{x}{2} e^{\frac{x}{2}}$

$u = x \qquad u' = 1$

$v = e^{\frac{x}{2}} \qquad v' = \frac{1}{2}e^{\frac{x}{2}}$

ii) $xe^{\frac{x}{2}} = \int e^{\frac{x}{2}}\,dx + \int \frac{x}{2} e^{\frac{x}{2}}\,dx$

$\frac{1}{2}\int xe^{\frac{x}{2}}\,dx = xe^{\frac{x}{2}} - \int e^{\frac{x}{2}}\,dx$

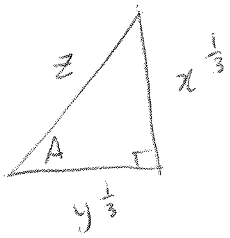
$\int xe^{\frac{x}{2}}\,dx = 2xe^{\frac{x}{2}} - 2 \cdot 2e^{\frac{x}{2}} + C$

$= 2(x-2)e^{\frac{x}{2}} + C$

Question 16

a) i) $\tan A = \sqrt[3]{\frac{x}{y}}$ $\quad 0 < A < \frac{\pi}{2}$

$= \frac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}}$




let $z$ be the hypotenuse

$(x^{\frac{1}{3}})^2 + (y^{\frac{1}{3}})^2 = z^2$ [Pythagoras' Thm]

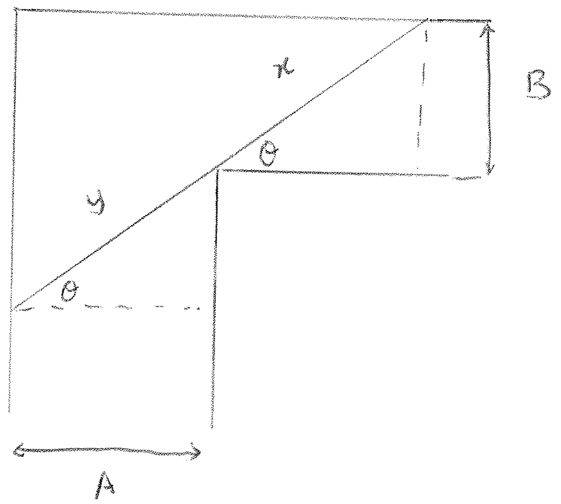
$\therefore z = \sqrt{x^{\frac{2}{3}} + y^{\frac{2}{3}}}$

as $0 < A < \frac{\pi}{2}$ $\quad \cos A > 0$

$\cos A = \frac{y^{\frac{1}{3}}}{\sqrt{x^{\frac{2}{3}} + y^{\frac{2}{3}}}}$ where $y > 0$.

ii) $\sin A = \frac{x^{\frac{1}{3}}}{\sqrt{x^{\frac{2}{3}} + y^{\frac{2}{3}}}}$ where $x > 0$

b)




Let $L = x + y$

$\sin\theta = \frac{B}{x}$

$x = B\operatorname{cosec}\theta$

$\cos\theta = \frac{A}{y}$

$\therefore y = A\sec\theta$

$\therefore L = B\operatorname{cosec}\theta + A\sec\theta$

ii) If $\theta = 0$ then pipe is completely in the corridor of width B and as $\theta \to 0$ then $L \to \infty$. Likewise if $\theta = \frac{\pi}{2}$ the pipe is completely in corridor with witdth A. So somewhere in the interval $0 < \theta < \frac{\pi}{2}$ is an angle that will minimise L. Any pipe larger than this L will not fit around the corner. Any pipe smaller than this will not be the longest pipe possible.

iii) $L = A\sec\theta + B\operatorname{cosec}\theta$

$$\frac{dL}{d\theta} = A\sec\theta\tan\theta - B\operatorname{cosec}\theta\cot\theta$$

$$= \frac{A\sin\theta}{\cos^2\theta} - \frac{B\cos\theta}{\sin^2\theta}$$

$$= \frac{A\sin^3\theta - B\cos^3\theta}{\sin^2\theta\cos^2\theta}$$

L is minimised when $\frac{dL}{d\theta} = 0$

i.e. when $A\sin^3\theta - B\cos^3\theta = 0$

$$A\sin^3\theta = B\cos^3\theta$$

$$\frac{\sin^3\theta}{\cos^3\theta} = \frac{B}{A}$$

$$\tan^3\theta = \frac{B}{A}$$

$$\tan\theta = \sqrt[3]{\frac{B}{A}}$$

$\therefore$ $\tan\theta = \sqrt[3]{\frac{B}{A}}$ will give the minimum solution.

iv) Since $L = A\sec\theta + B\operatorname{cosec}\theta$

$$= \frac{A}{\cos\theta} + \frac{B}{\sin\theta}$$

when $\tan\theta = \sqrt[3]{\frac{B}{A}}$

$$\cos\theta = \frac{A^{\frac{1}{3}}}{\sqrt{A^{\frac{2}{3}} + B^{\frac{2}{3}}}}$$ from part a(i)

and $$\sin\theta = \frac{B^{\frac{1}{3}}}{\sqrt{A^{\frac{2}{3}} + B^{\frac{2}{3}}}}$$ from part a(ii)

then $$L = \frac{A\sqrt{A^{\frac{2}{3}} + B^{\frac{2}{3}}}}{A^{\frac{1}{3}}} + \frac{B\sqrt{A^{\frac{2}{3}} + B^{\frac{2}{3}}}}{B^{\frac{1}{3}}}$$

$$= A^{\frac{2}{3}}\left(A^{\frac{2}{3}} + B^{\frac{2}{3}}\right)^{\frac{1}{2}} + B^{\frac{2}{3}}\left(A^{\frac{2}{3}} + B^{\frac{2}{3}}\right)$$

$$= \left(A^{\frac{2}{3}} + B^{\frac{2}{3}}\right)\left(A^{\frac{2}{3}} + B^{\frac{2}{3}}\right)^{\frac{1}{2}}$$ [factorising out $(A^{\frac{2}{3}} + B^{\frac{2}{3}})$]

$$= \left(A^{\frac{2}{3}} + B^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

c) Consider $(1,2)$ $(1,2,2)$ $(1,2,2,2)$ ...

2 terms 3 terms 4 terms

Suppose the $(n-1)^{th}$ group is $(1, \underbrace{2,2,2, \ldots 2}_{n-1 \text{ terms}})$

Then $2+3+4+\ldots+n \leq 1234$

$\therefore \frac{n-1}{2}\left[4+(n-2)\right] \leq 1234$

$n^2+n-2 \leq 2468$

$n(n+1) \leq 2470$

we can see that when $n=49$ $49 \times 50$ is close to $2470$.

$\therefore$ $n=49$ goes up to the $48^{th}$ group.

$1234 - \frac{48 \times 49}{2} = 10$

$\therefore$ the $49^{th}$ group can only have 1 1's and 9 2's

There are 49 1's in the first 1234 terms

and $(1+2+3+\ldots+48)+9$ 2's in the first 1234 terms.

$= \left(\frac{48}{2}\left[2+47\right]+9\right)$ 2's

$= 1185$ 2's.

$\therefore$ sum of first 1234 terms is

$49 + (1185 \times 2)$

$= 2419$.